

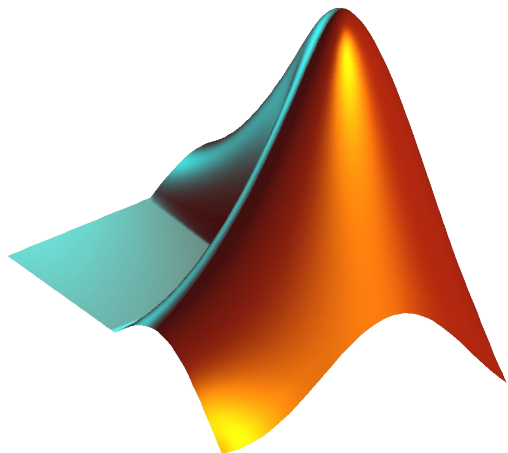
CS 1112 Introduction to Computing Using MATLAB

Instructor: Dominic Diaz

Website:

<https://www.cs.cornell.edu/courses/cs1112/2022fa/>

Today: sorting/searching



Agenda and announcements

- Last time
 - Recursion
- Today
 - An algorithm for sorting (merge sort)
 - An algorithm for searching (binary search)
- Announcements
 - Project 6 due Dec 5th
 - Code 'til you drop session on Dec 14th
 - Last discussion exercise this week
 - Optional but can replace your lowest discussion score.
 - Final exam on Thursday, December 15th from 2 - 4:30 PM in Olin 155
 - Check your “final exam time and location” CMS assignment if you have an SDS letter. If you have 3+ finals in a 24 hour period, submit a regrade request and we can reschedule this exam.

Searching for an item in an unorganized collection?

- May need to look through the whole collection to find the target item
- Could use linear search

For example, find value x
in vector v

x	1				
v	5	4	9	2	1

```
% Linear Search
% f is index of first occurrence
% of value x in vector v.
% f is -1 if x not found.
k= 1;
while k<=length(v) && v(k)~=x
    k= k + 1;
end

if k>length(v)
    f= -1; % signal for x not found
else
    f= k;
end
```

In the worst case,
the while loop will
be evaluated n
times, assuming
 $n=length(v)$.

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if k>length(v)
    f= -1; % signal f
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```

In the worst case,
the while loop will
be evaluated n
times, assuming
 $n=length(v)$.

Searching in a
sorted list should
require less work!

An ordered (sorted) list

Manhattan phone book has
1,000,000+ entries

If the list were unsorted, you would
have to look through every name
to find someone.

07 566-1282	Cartage New England Inc 26 Allen Ln Ipswich 01938.....	978 356-9960	Carter F 34 Hillock Ros 02131.....	617 327-1105	Carter Nella E 333 Massachusetts Av Bos 02115.....	617 267-6483
81 447-4101	Cartagema Lydia 18 Jewett Ros 02131.....	617 323-7639	Faye & Ricky 357 Columbus Av Bos 02116.....	617 437-7331	Nicholas S F 115 Randolph Av MI 02186.....	617 698-5307
00 257-9981	Cartagena Avith 9 Bancroft Row 02119.....	617 442-9780	Francis S 134 Temple W Ros 02132.....	617 323-6781	Nick 21 Fairfield Bos 02116.....	617 267-5222
07 566-1282	B Hjd 02136.....	617 361-5253	Franklin & Anne 221 Mt Auburn Cam 02138.....	617 354-0798	Nick & Debbi 196 Herrick Rd Newton 02459.....	617 527-0480
07 364-5188	Jessica 50 Decatur Cha 02129.....	617 241-0152	Fred 42 Howland Rd 02138.....	617 524-3078	Nicole 146 Hutchings Row 02121.....	617 698-0713
361-0380	Lucilla 174 Harvard Cam 02139.....	617 491-5621	Fred 96 Hinchley Rd MI 02186.....	617 698-1343	Norman G 38 Chickatabow Dor 02122.....	617 822-1203
07 566-4548	M 95 Rowe Ros 02131.....	617 323-9713	G & R 8 Verdon Dor 02124.....	617 436-8906	P 94 Crestwood Pk Ros 02121.....	617 427-4754
07 628-8248	Melvin 501 Green Cam 02139.....	617 576-1061	G T 27 Franklin Av Som 02145.....	617 623-7121	P E 501 E Sixth S Bos 02127.....	617 268-4213
07 445-5116	Carte Nicholas 18 Appleton Boston 02116.....	617 695-6996	Gayle 25 Frontenac Dor 02124.....	617 825-0322	P L 44 Hutchings Row 02121.....	617 427-9170
07 822-2982	Cartegena O 1 Millard Bos 02118.....	617 338-8219	Geo S 115 Moss Hill Rd Jam 02130.....	617 522-3215	P R 91 Bymer Jam 02130.....	617 983-8692
07 569-2698	Carten Thos J Sr & Claire 1 Paradise Rd MI 02186.....	617 698-6163	George 125 Nashes Bos 02114.....	617 367-9548	Paul & Constance 114 Anawan Av W Ros 02130.....	617 325-2036
07 667-5190	Thomas & Kathleen 50 Thompson Ln MI 02186.....	617 696-6919	Carter Halliday Associate 107 S Street Bos 02111.....	617 456-1689	Paul E 501 E Sixth S Bos 02127.....	617 268-4546
07 569-1417	Carte A Ros 02131.....	617 327-2257	Carter Harry F 26 Ransing Bk Rd W Ros 02132.....	617 325-5465	Paul M 27 Union Bk 02135.....	617 787-2115
07 338-9110	A Rosbury.....	617 442-5230	Carter Hide Co Inc 146 Summer Bos 02110.....	617 542-7987	Carter Pile Driving Inc 17 Beaver Ct Framingham 01702.....	781 235-8488
07 825-9195	A 31 Bethune Wy Rosbury 02119.....	617 442-1219	Carter Hilary 61 Harvey Cam 02140.....	617 876-2750	Carter Prudence 46 Franklin Watertown 02172.....	617 393-3782
07 296-1593	A 260 Putnam Av Cambridge 02139.....	617 492-4174	Horace 241 Walnut Av Rosbury 02119.....	617 442-5307	Prudence 46 Franklin Watertown 02172.....	617 926-7063
07 670-2078	A M 255 Massachusetts Av Bos 02115.....	617 266-7153	Howard Jr 26 Notre Dame Ros 02119.....	617 445-5552	Reginald 106 Brunswick Dorchester 02122.....	617 541-2843
07 623-9001	Adams 361 Centre St MI 02186.....	617 698-9074	J Cam.....	617 354-2688	Renee & Andrew 30 Walnut Bos 02108.....	617 720-3765
07 296-4725	Alice 108 Kilmarnock Bos 02115.....	617 425-0193	J 15 Chatham Bro 02446.....	617 232-7990	Carter Rice Dowd Bulkley Dutton Publishing 163 Main Wilmington 01887	800 638-1671
07 542-1521	Alice 45 Market Cambridge 02139.....	617 945-2711	J 518 Harvard Bro 02446.....	617 730-9483	Toll Free-Dial '2' & Then.....	800 619-7447
07 364-5232	Andrew F 62 Vinal Av Som 02143.....	617 625-7623	J 775 Vhu Pkwy West Rosbury 02132.....	617 323-5574	Toll Free-Dial '2' & Then.....	800 648-7447
07 541-5649	Carte Anne MD 1101 Beacon Bro 02446.....	617 739-1022	Carter J Jacques MD 1 Brookline Pl Bro 02446.....	617 735-8787	Headquarters 613 Main Wilmington 01887	978 988-7447
07 739-2662	Carte Athens 272 Newbury Boston 02116.....	617 536-6329	Carter J M 1410 Columbia Rd S Bos 02127.....	617 464-1040	Ingalis Cronin 163 Main Wilmington 01887	800 638-1673
07 879-0030	B E 48 Gladstone Av Mat 02126.....	617 296-6911	Carter J M Ornamental Ironworks Call.....	617 436-5353	Toll Free-Dial '2' & Then.....	800 638-1673
07 436-1513	Carte Barbara L MD Tufts-New England Medical Center Bos 02111	617 636-0051	Carter J Veal Co 48 Newmarket Sq Ros 02118.....	617 442-1775	Toll Free-Dial '2' & Then.....	800 648-7447
07 569-4119	Carte Becky Bos 02114.....	617 523-4368	Carter James 1573 Cambridge St Cam 02138.....	617 492-1214	Call.....	978 988-7447
07 569-8762	Bernard J 112 Gladstone E Bos 02128.....	617 567-3430	James 182 Fisher Av Rosbury 02120.....	617 739-2193	Call.....	978 988-7447
07 879-0030	Bithiah 25 Medway Dor 02124.....	617 298-8713	James 37 Gold Star Rd Cambridge 02140.....	617 876-8841	Carter Richard 1079 Cornwell Av Brighton 02215.....	617 987-0836
07 541-5649	Blake 26 Mt Vernon Bos 02108.....	617 367-9931	Jas L 14 Roseberry Rd Mat 02126.....	617 361-0773	Richard A 97 Mt Vernon Bos 02108.....	617 566-7293
07 879-0030	Carte Broadcasting Co 20 Park Pl Bos 02116.....	617 423-0210	Jane 114 Adena Rd Newton 02460.....	617 964-0435	Carter Richard A MD 170 Cornwell Av Bos 02116.....	617 267-0710
07 541-5649	Carte & Burgess Consultants Inc 23 East St Cam 02141.....	617 225-0200	Jeffrey 41 Warren Av Bos 02116.....	617 426-5994	15 Mercer S Bos 02127.....	617 268-0448
07 541-5649	Carter C 2000 Cornwell Av Bri 02135.....	617 782-2118	John 11 Mansfield Bri 02134.....	617 987-2163	Robert L 175 Rickdale Av Cam 02140.....	617 864-1535
07 569-4119	C 228 Faywood Av East Boston 02128.....	617 569-1545	John 327 Summer Bos 02110.....	617 423-4334	Roger 150 St Botolph Bos 02115.....	617 424-6148
07 569-4119	C 359 Harvard Cam 02138.....	617 491-4822	John 40 Westwind Rd Dor 02125.....	617 282-1235	Roy 44 Concord Av Cam 02138.....	617 491-6115
07 569-4119	C 610 Wall Hill Mat 02126.....	617 296-6392	June O 329 A Summit Av Bri 02135.....	617 734-6109	Royce 18 Seminary Cha 02129.....	617 261-0418
07 569-8762	C & M 43 Burroughs Jam 02130.....	617 524-9558	K 38 Browning Av Dorchester 02124.....	617 265-8456		
			K 17 Edmond Dorchester 02127.....	617 282-1593		

There are many algorithms for sorting

- Merge sort (discussed this lecture)
 - Selection sort (exercise this week)
 - Insertion sort (discussed next lecture)
 - Bubble sort (read insight section 8.2)
 - Quick sort (a variant used by MATLAB's built-in sort function)
-
- Each has advantages and disadvantages. Some algorithms are faster (time efficient) while others are memory-efficient.
 - Great opportunity for learning to analyze the efficiency of programs and algorithms

Merge sort motivation

If I have two helpers, I would:

- Give each helper half the array to sort
- Then I get back the sorted subarrays and merge them

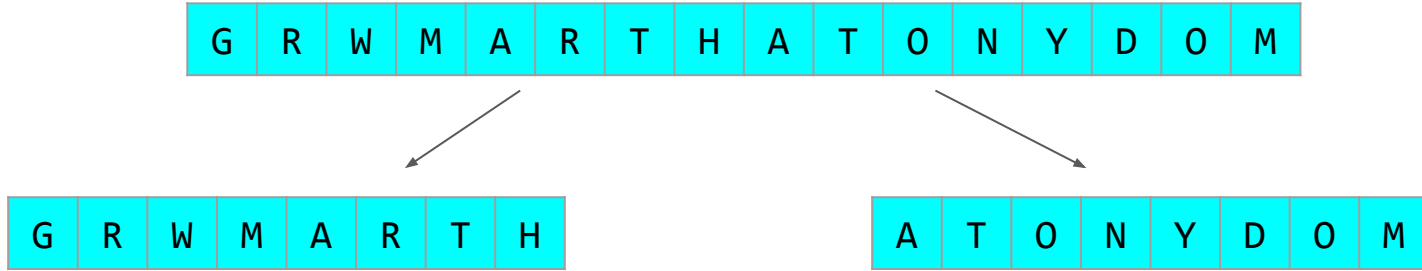
What if those two helpers each had two sub-helpers?

And each of those sub-helpers each has two sub-sub-helpers? And...

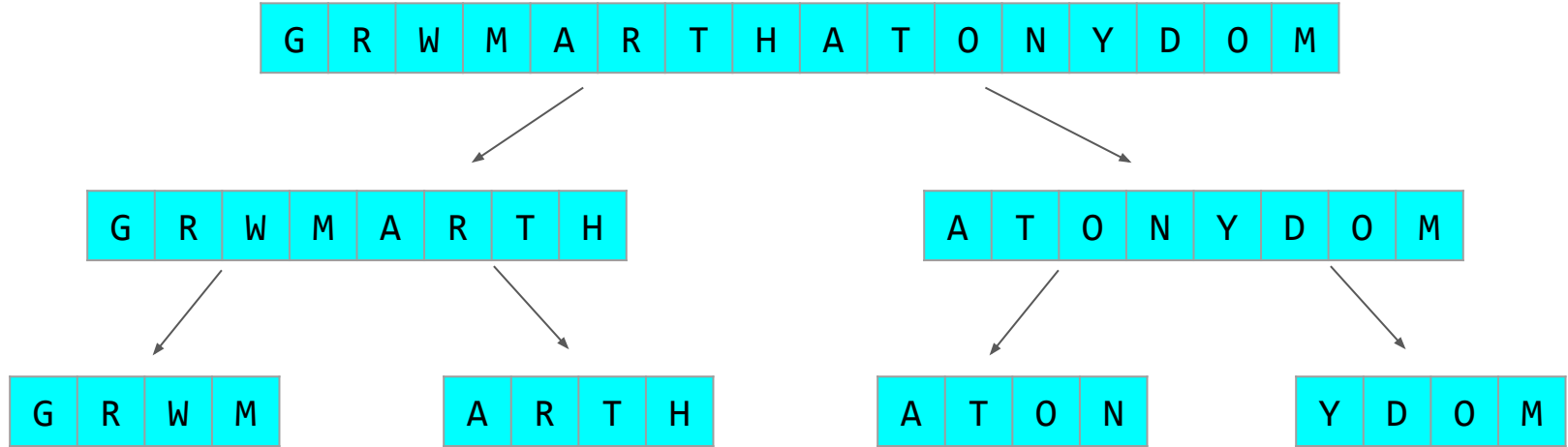
Let's see how we can apply this idea to sort an unsorted list

G	R	W	M	A	R	T	H	A	T	O	N	Y	D	O	M
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Let's see how we can apply this idea to sort an unsorted list

G R W M A R T H A T O N Y D O M

G R W M A R T H

A T O N Y D O M

G R W M

A R T H

A T O N

Y D O M

G R

W M

A R

T H

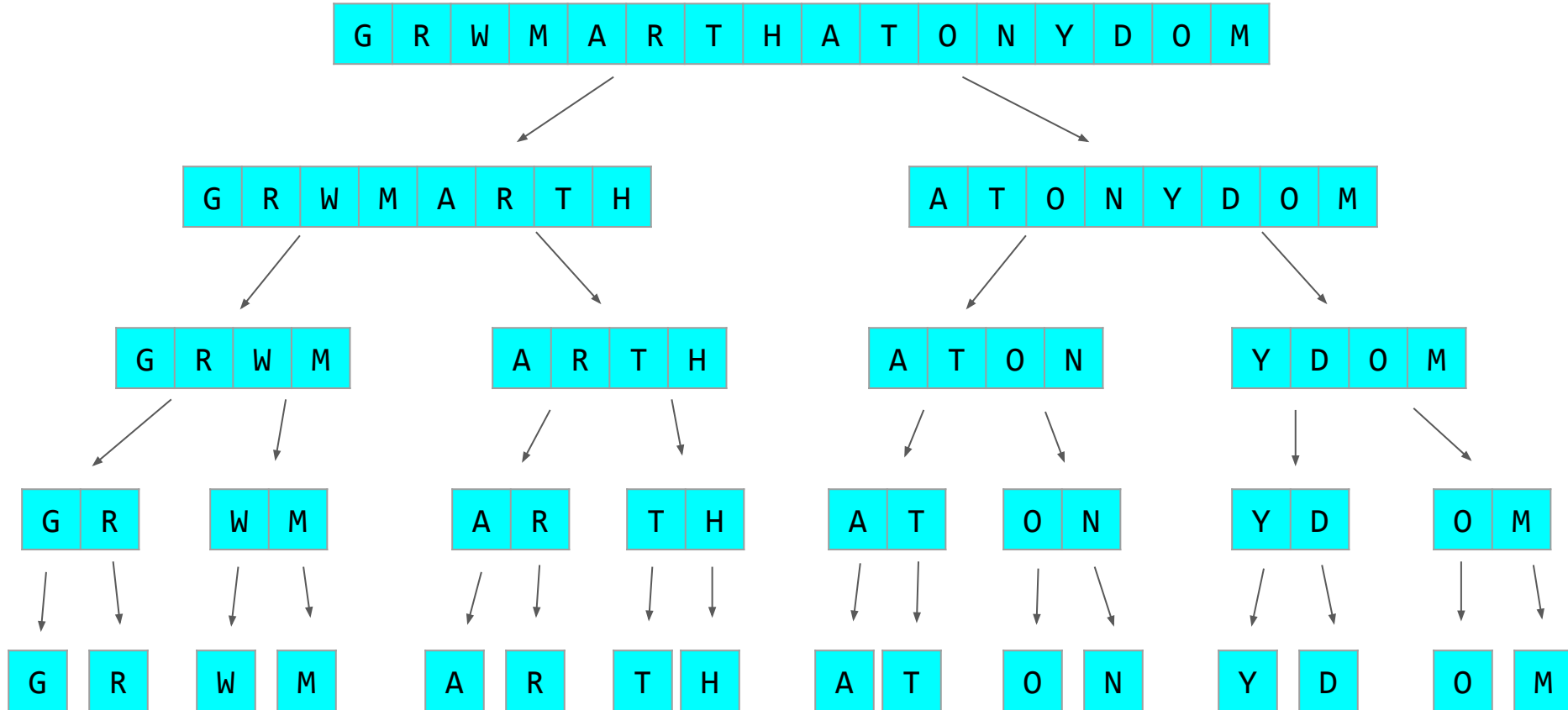
A T

O N

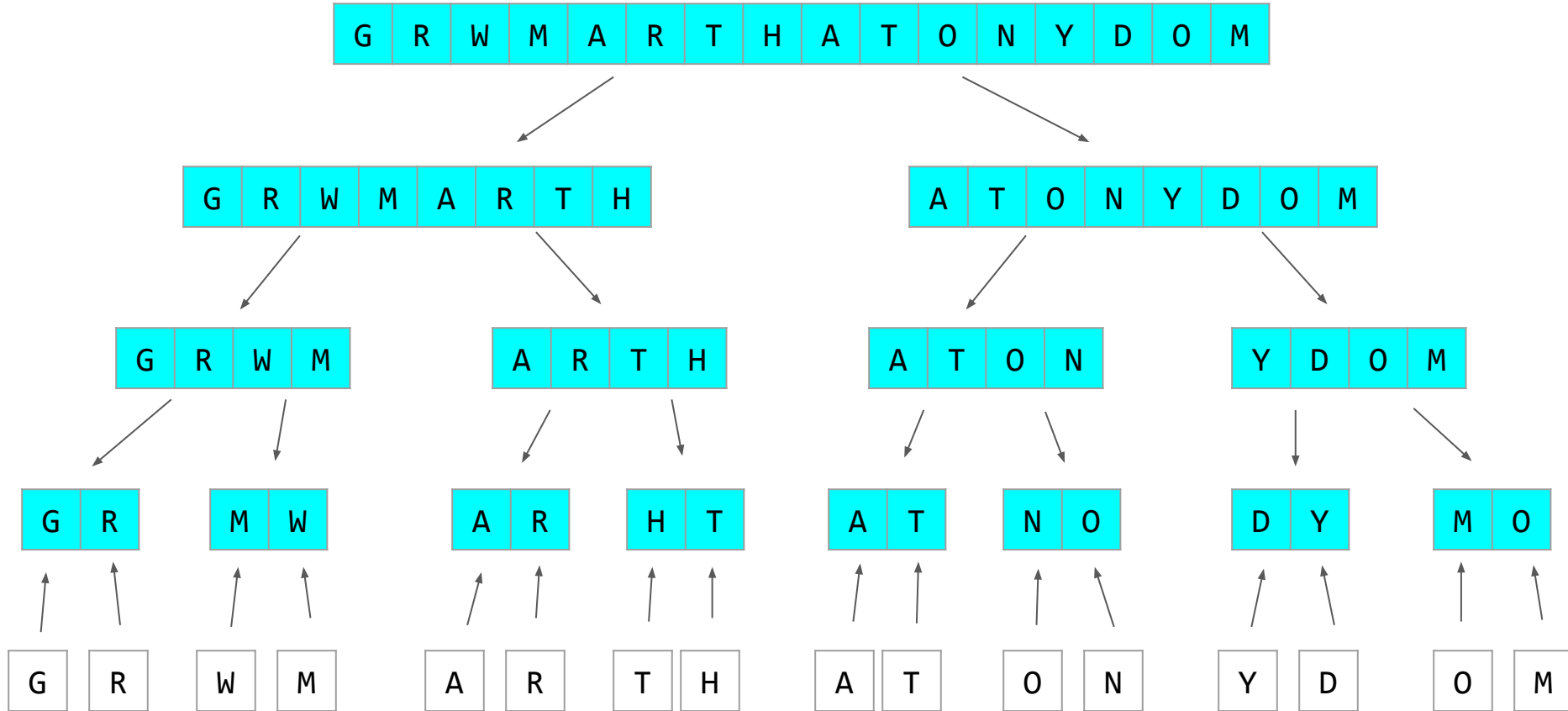
Y D

O M

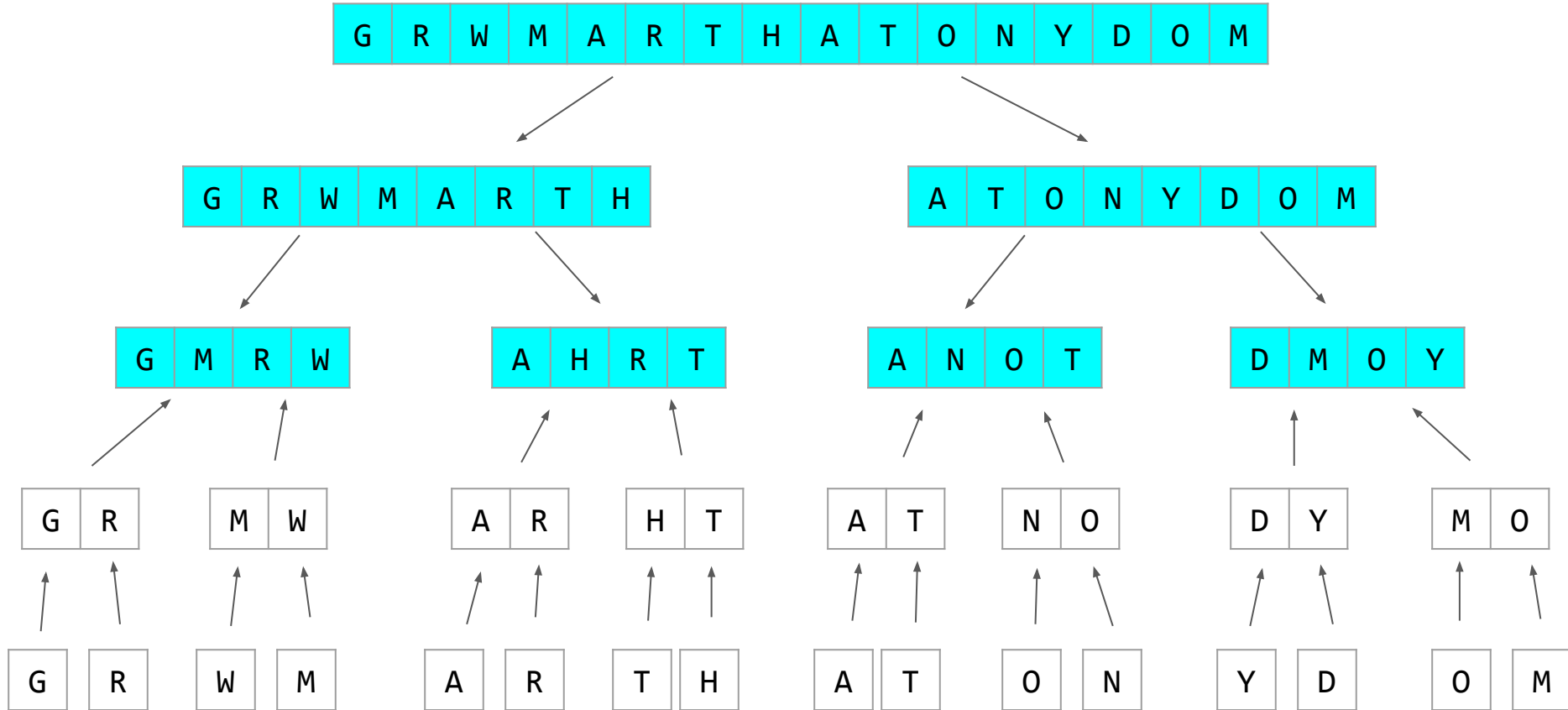
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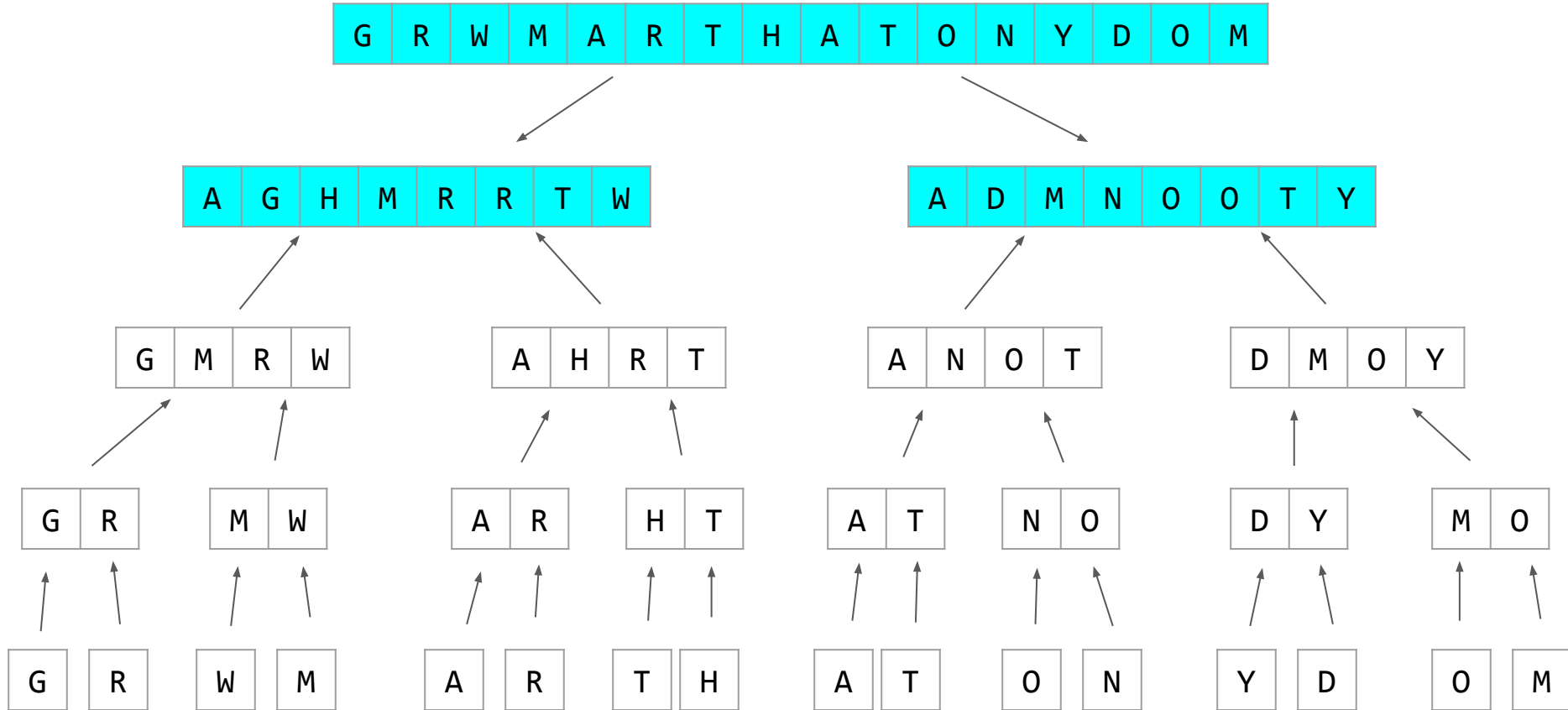
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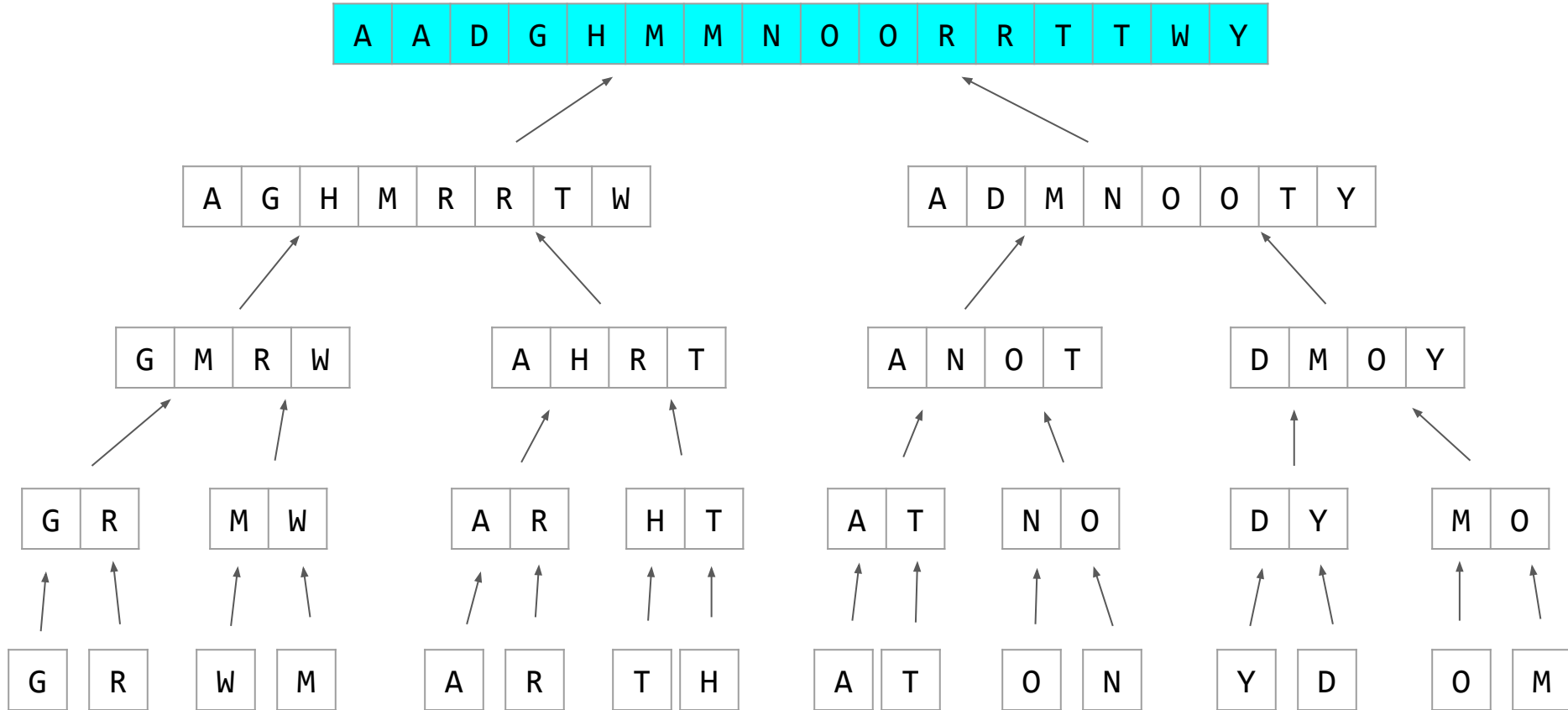
Let's see how we can apply this idea to sort an unsorted list



Let's see how we can apply this idea to sort an unsorted list



Let's see how we can apply this idea to sort an unsorted list



Merge sort - writing the code

```
function y = mergeSort(x)
% x is a vector. y is a vector
% consisting of the values in x
% sorted from smallest to largest.
n = length(x);
if n == 1
    y = x;
else
    m = n/2;
    yL = mergeSort(x(1:m));
    yR = mergeSort(x(m+1:n));
    % merge sorted yL and yR
    y = merge(yL,yR);
end
```

Using recursion we need to identify **base case(s)** and **make progress toward a base case**.

```
% sort vector x
if length(x) is 1,
    Do nothing. x is already sorted.
else
    Cut x in half
    Sort the left half
    Sort the right half
    Merge the sorted left and right halves
```

Base case

Make progress
towards base
case

Writing the merge sort function

```
function y = mergeSort(x)
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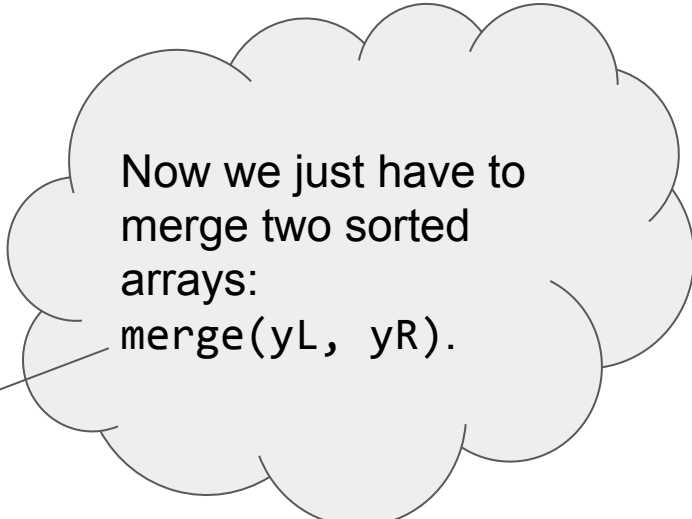
Assuming the function merge correctly merges two sorted arrays, are there any errors in this code?

If n is odd, m will be a decimal number.
Answer: Let's floor (or ceil) m

If x = [], your code enters infinite recursion.
Answer: Add to base cases

Writing the merge sort function

```
function y = mergeSort(x)
% x is a vector. y is a vector
% consisting of the values in x
% sorted from smallest to largest.
n = length(x);
if n == 1 || n == 0
    y = x;
else
    m = floor(n/2);
    yL = mergeSort(x(1:m));
    yR = mergeSort(x(m+1:n));
    % merge sorted yL and yR
    y = merge(yL,yR);
end
```



Now we just have to
merge two sorted
arrays:
merge(yL, yR).

How can we merge two sorted arrays?

L

12	33	35	45
----	----	----	----

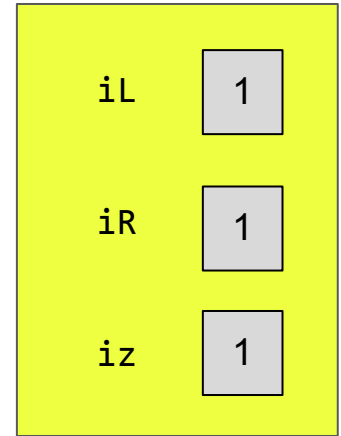
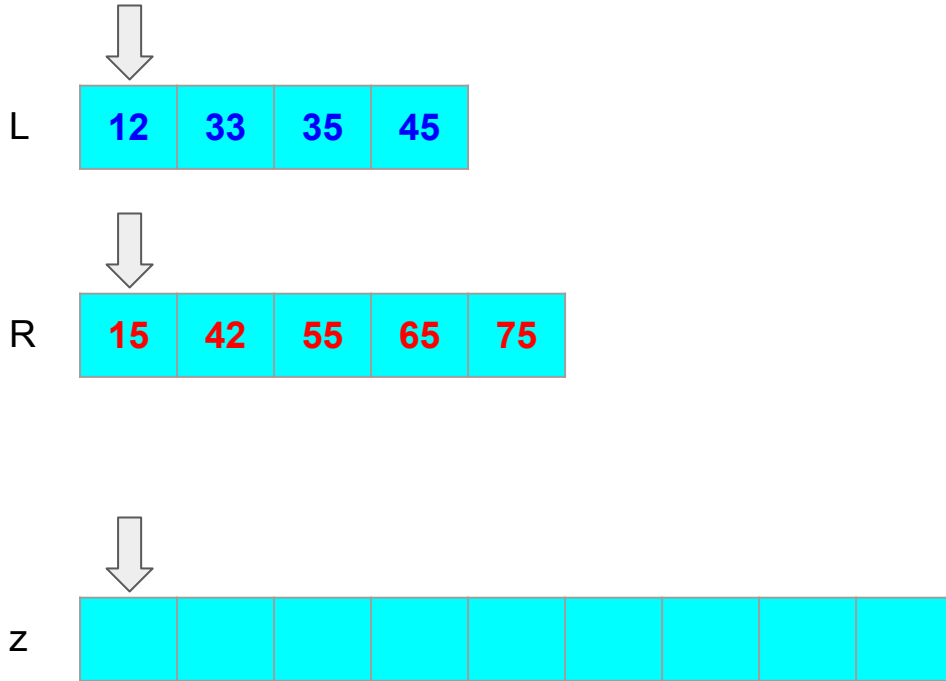
R

15	42	55	65	75
----	----	----	----	----

Z

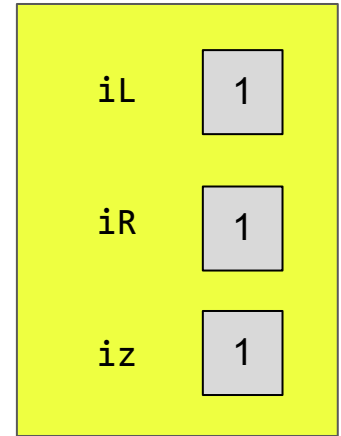
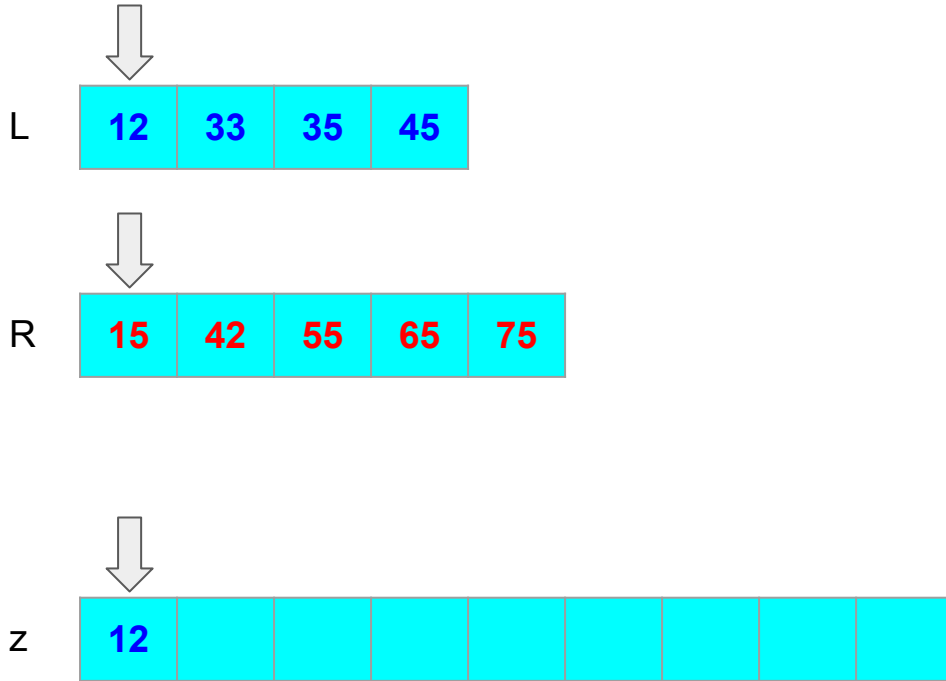
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How can we merge two sorted arrays?



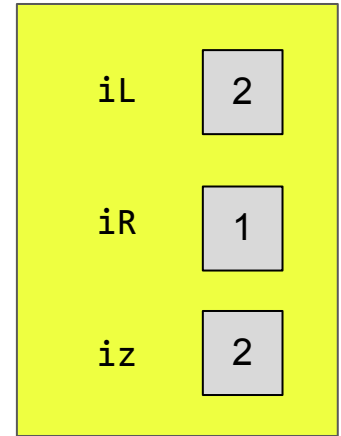
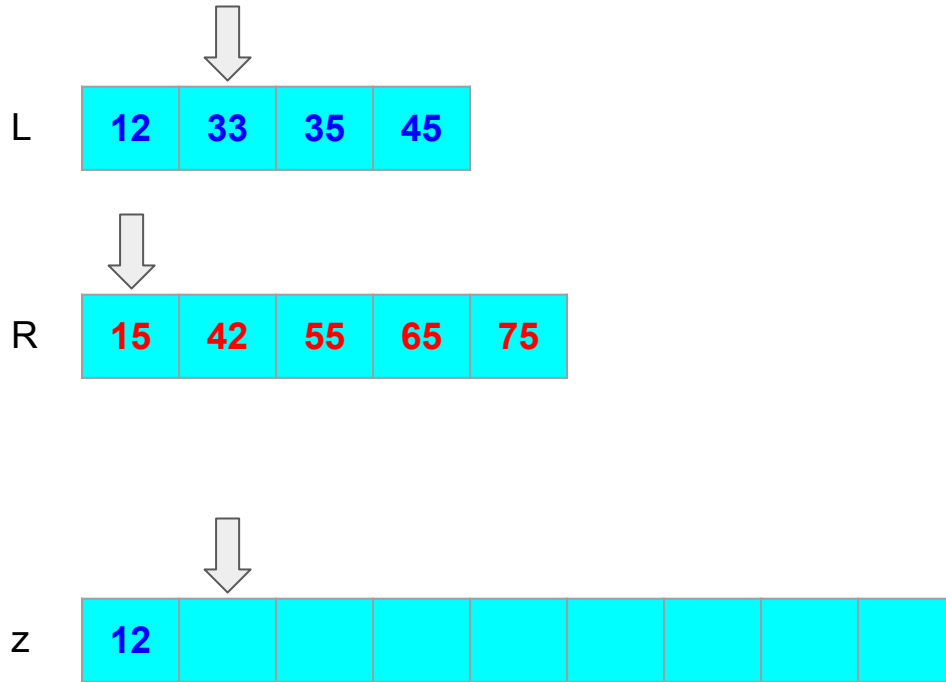
```
if L(iL) <= R(iR)
    z(iz) = L(iL);
    Increment iL and iz;
end
```

How can we merge two sorted arrays?



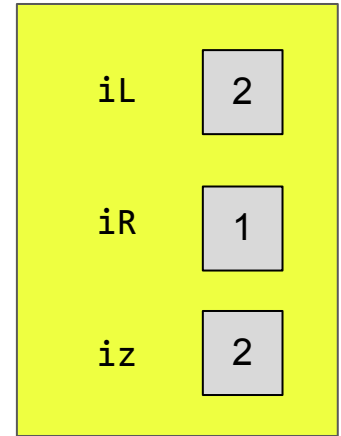
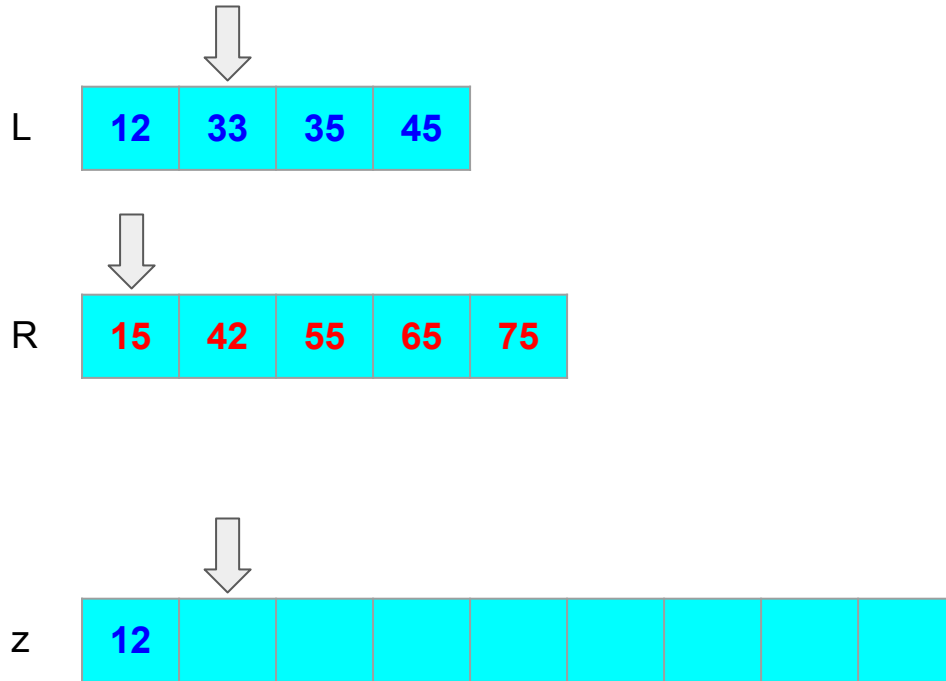
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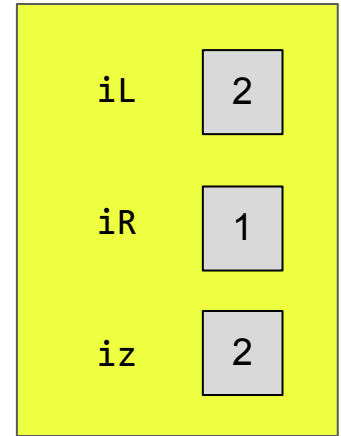
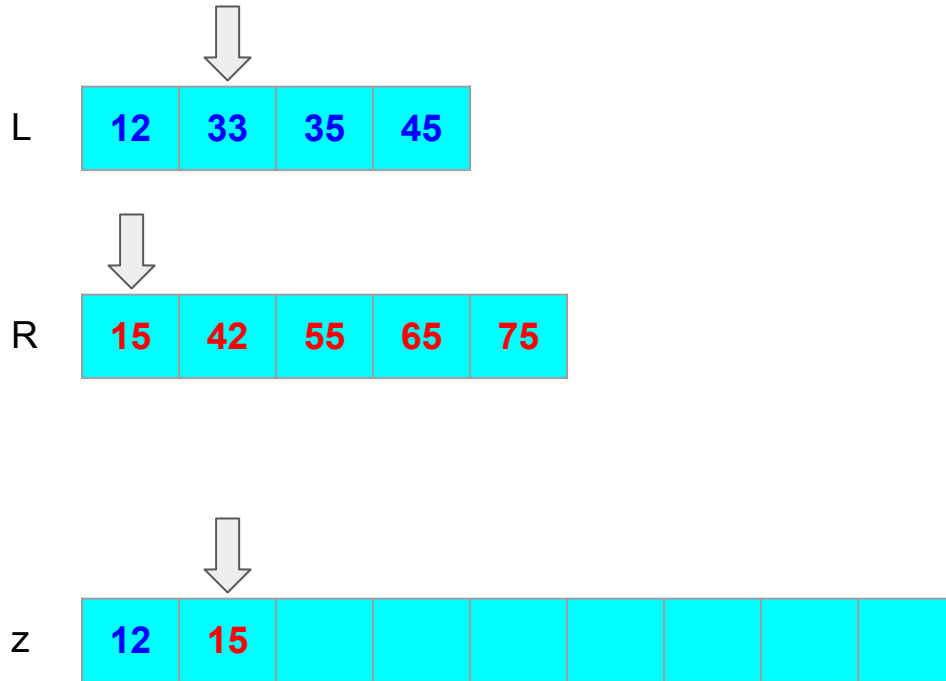
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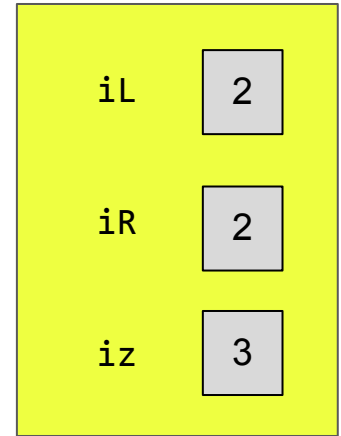
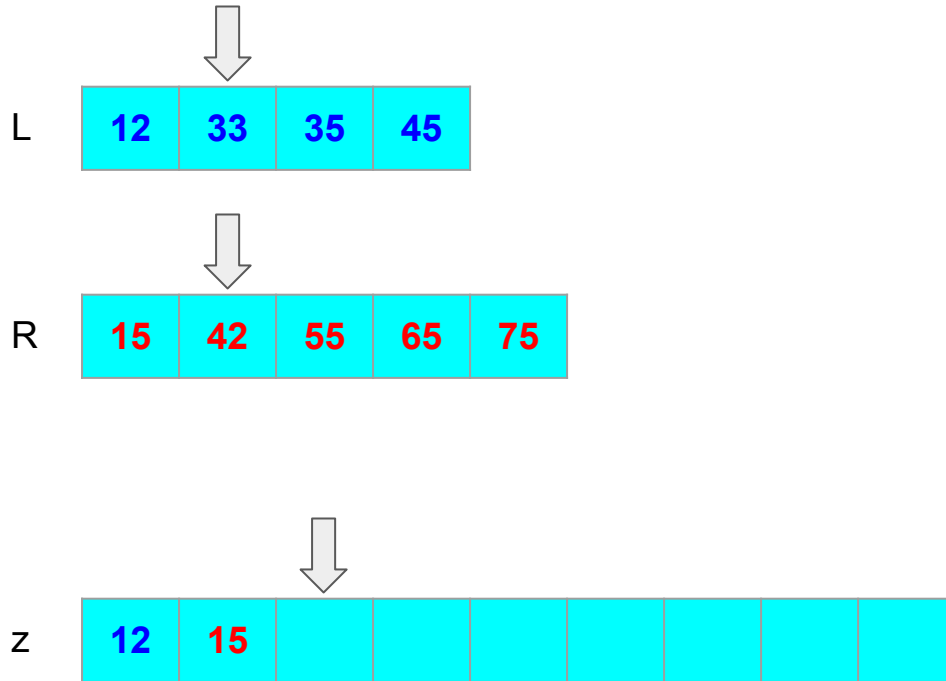
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How can we merge two sorted arrays?



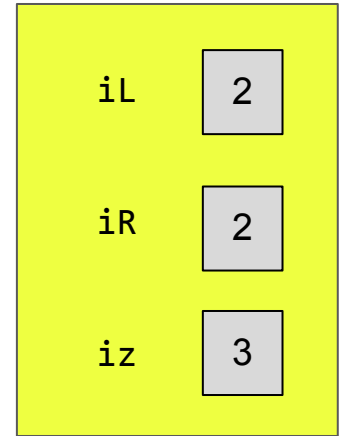
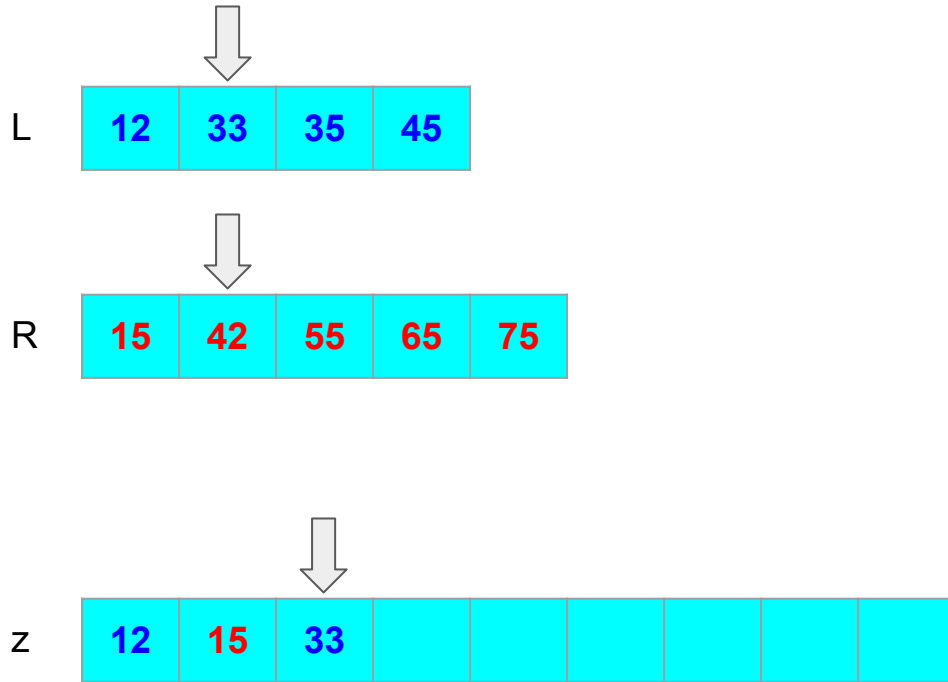
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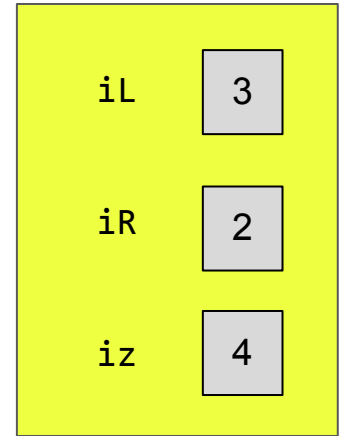
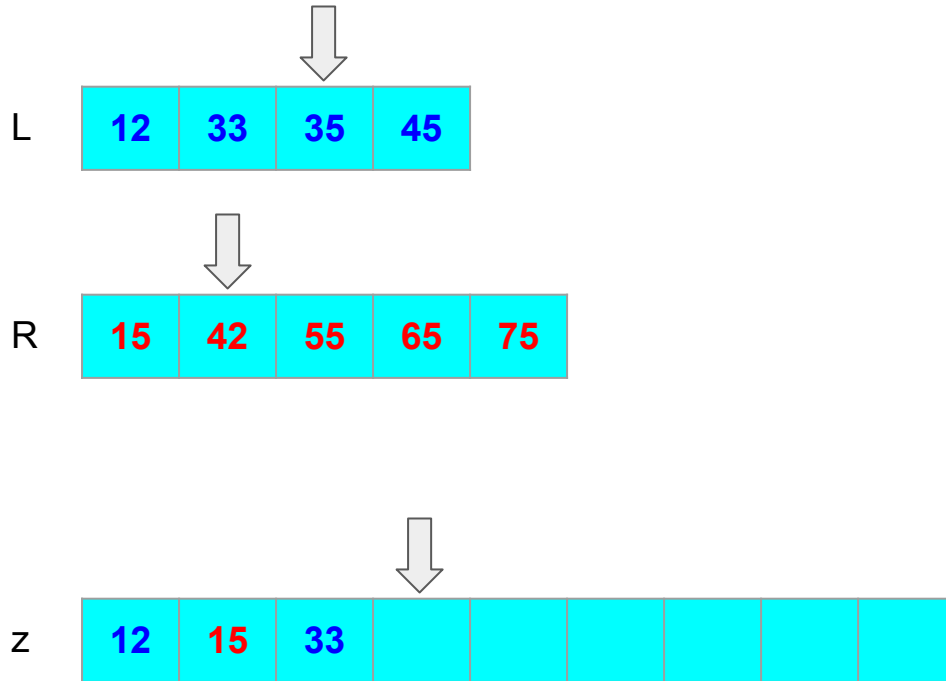
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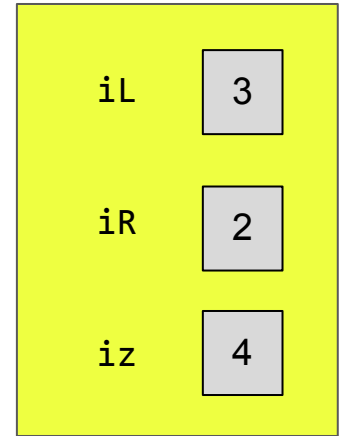
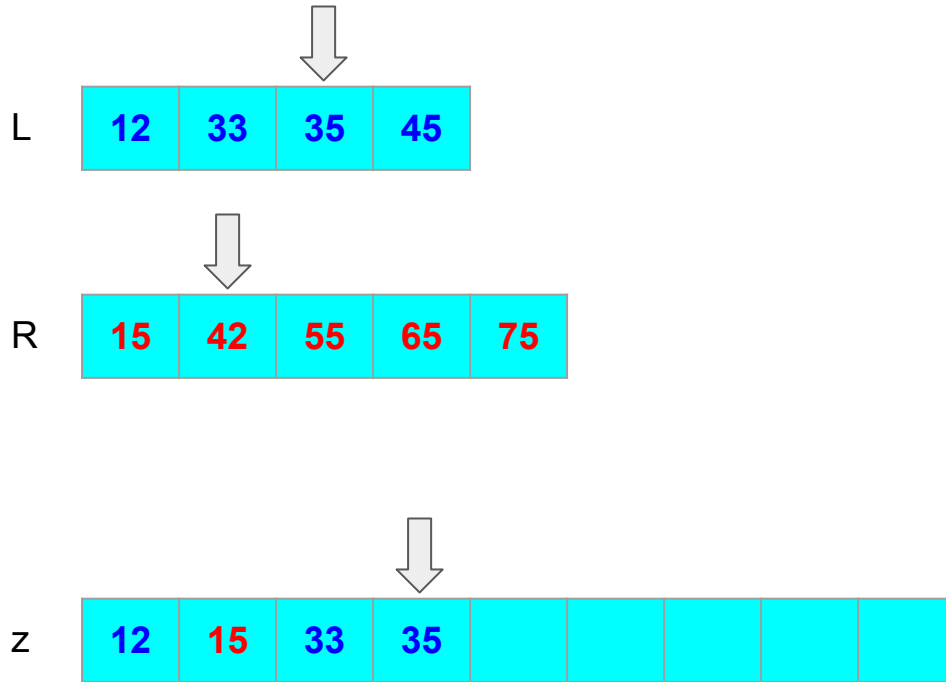
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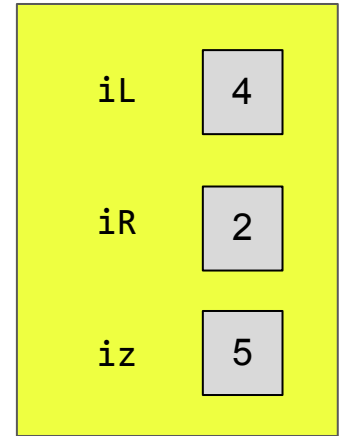
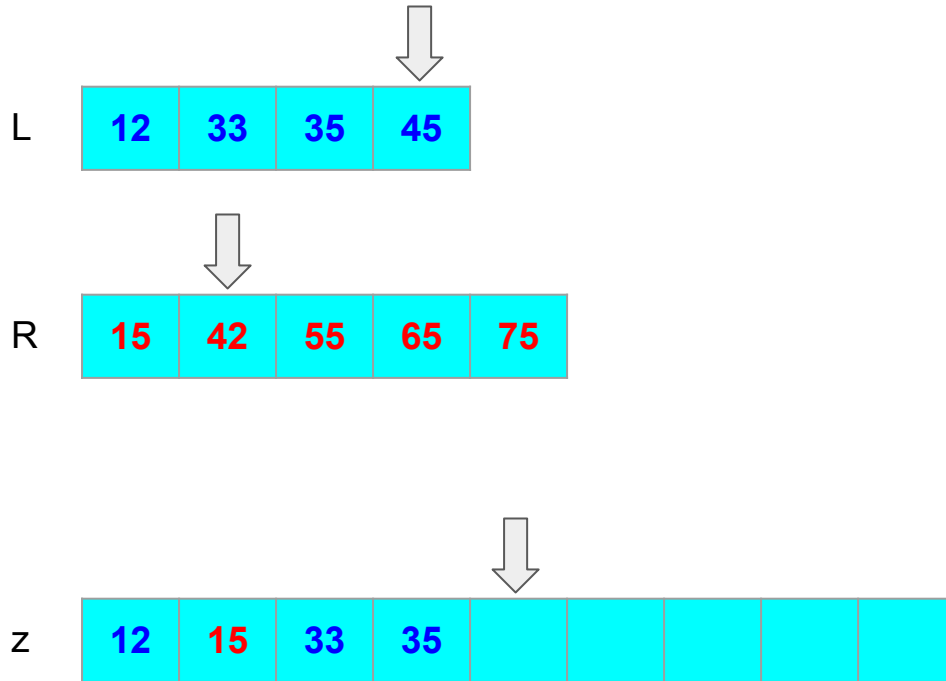
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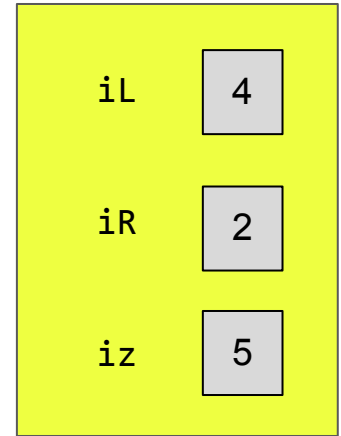
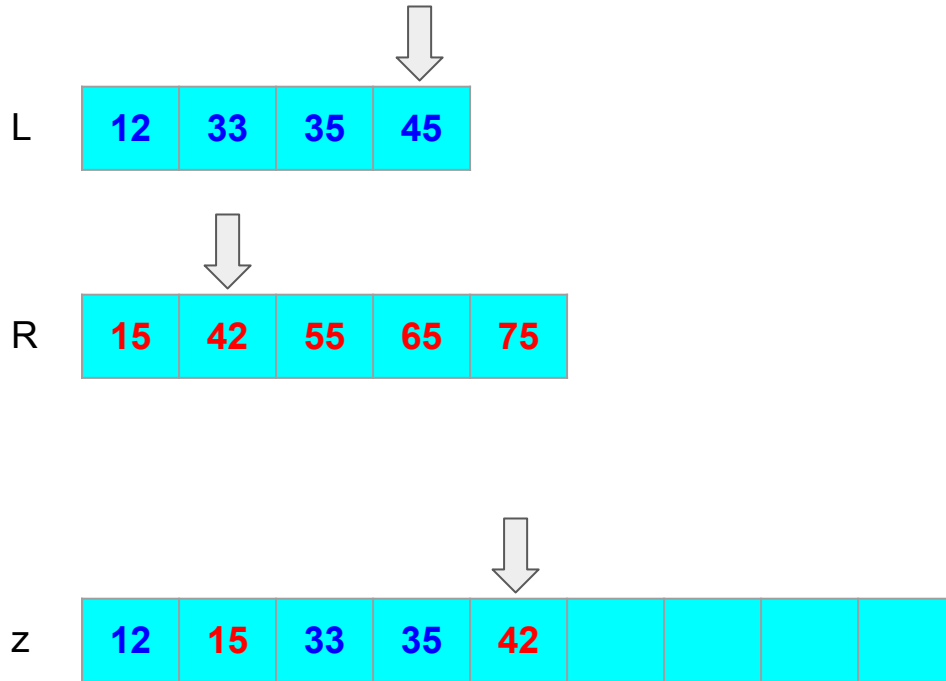
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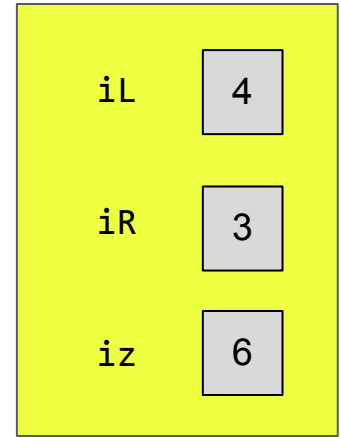
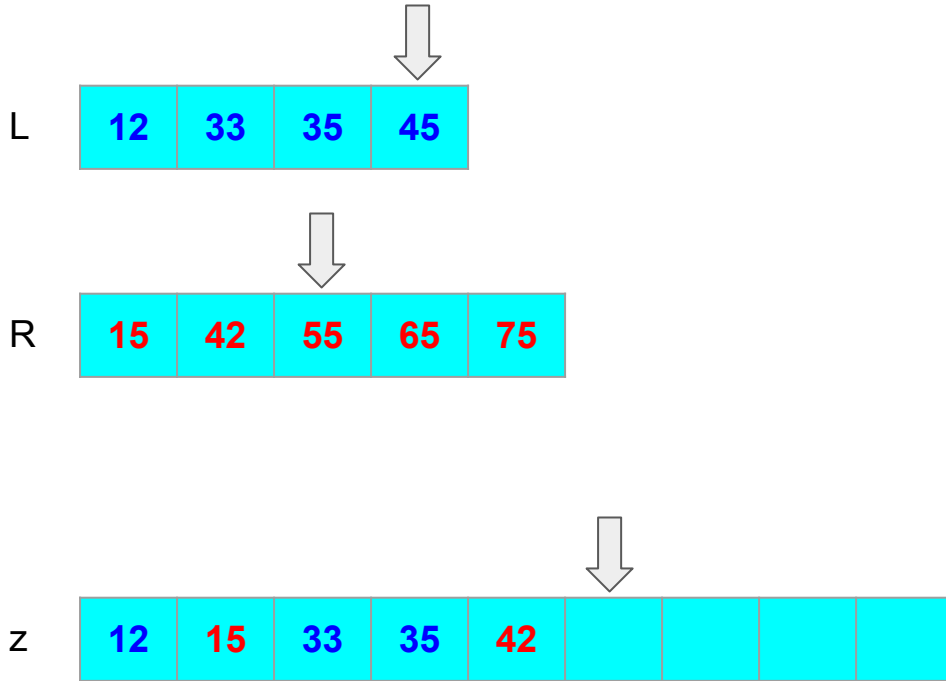
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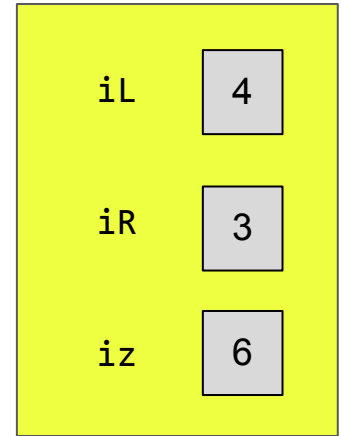
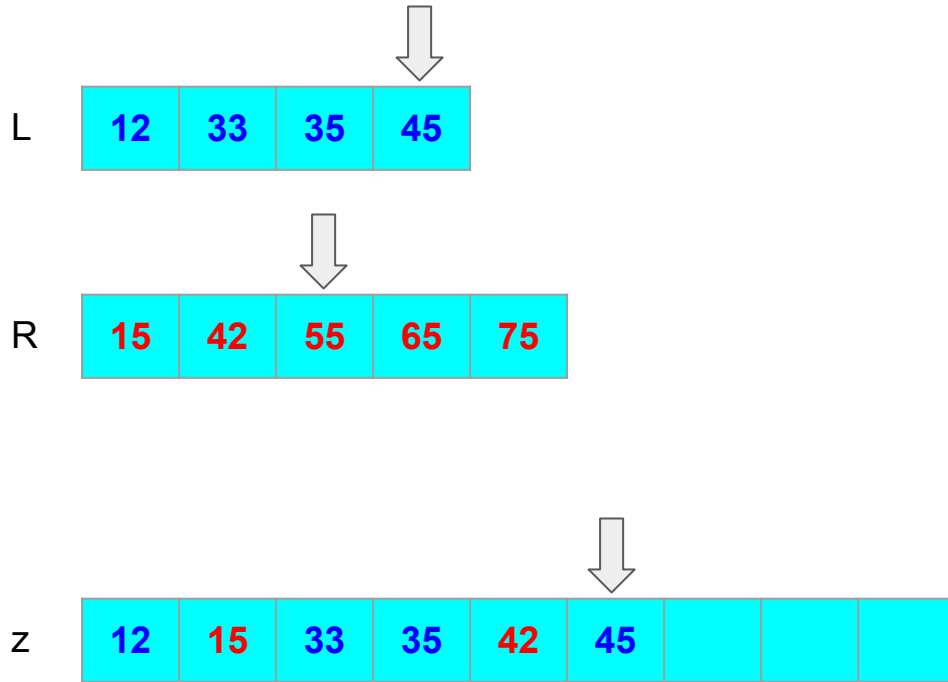
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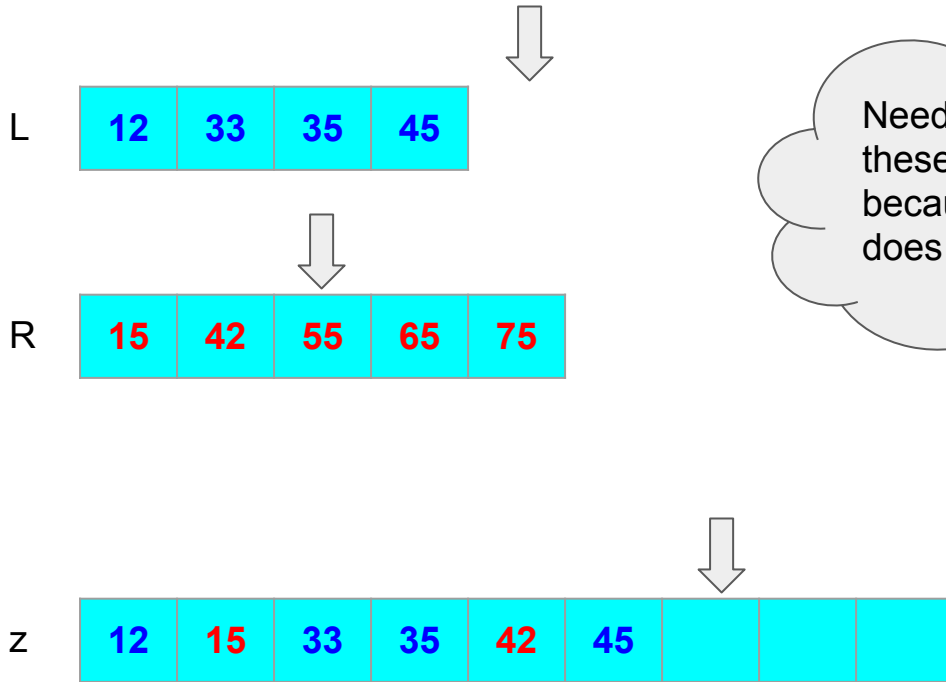
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```


How can we merge two sorted arrays?

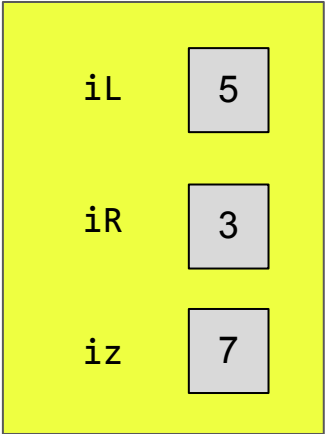


```
if L(iL) <= R(iR)
    z(iz) = L(iL);
    Increment iL and iz;
else
    z(iz) = R(iR);
    Increment iR and iz;
end
```

How can we merge two sorted arrays?

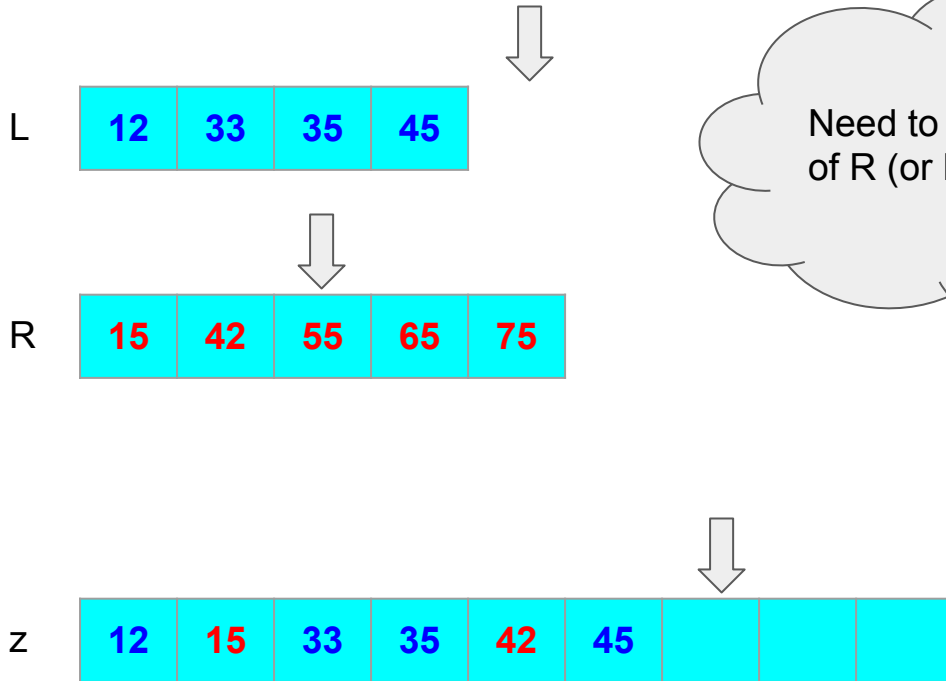


Need to stop doing these comparisons because $L(iL) = L(5)$ does not exist.



```
if L(iL) <= R(iR)
    z(iz) = L(iL);
    Increment iL and iz;
else
    z(iz) = R(iR);
    Increment iR and iz;
end
```

How can we merge two sorted arrays?

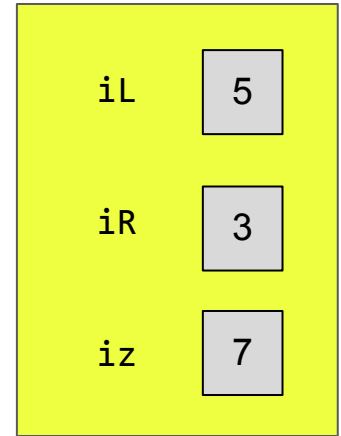
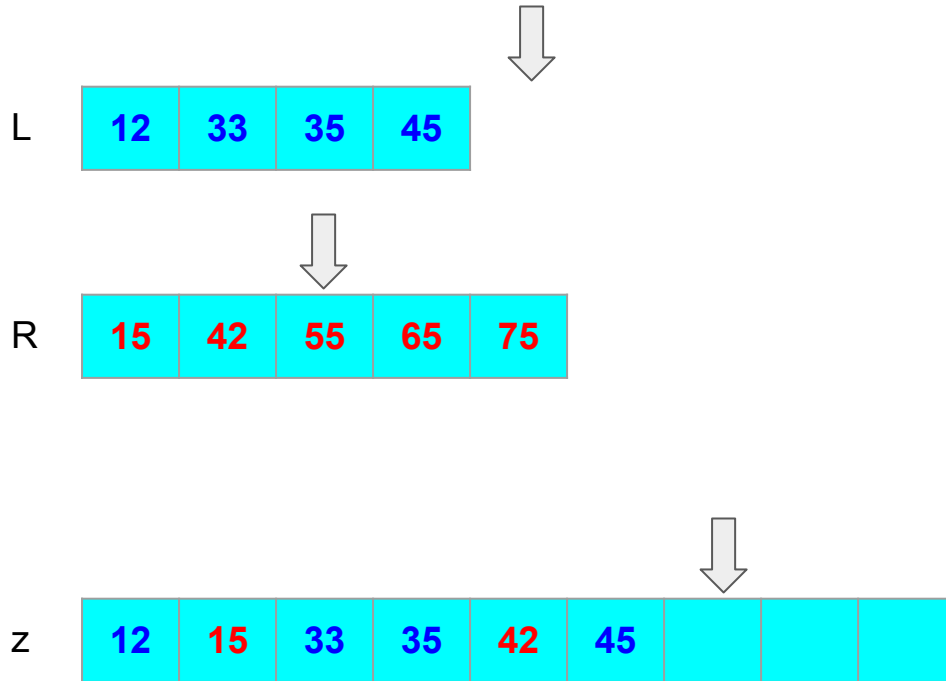


Need to fill in the rest
of R (or L) into z

iL	5
iR	3
iz	7

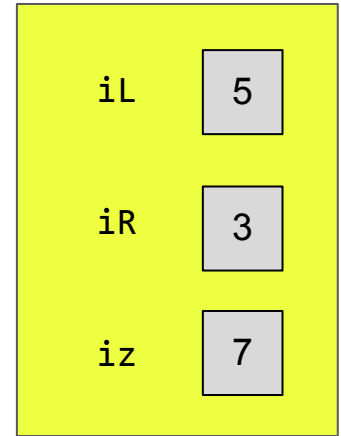
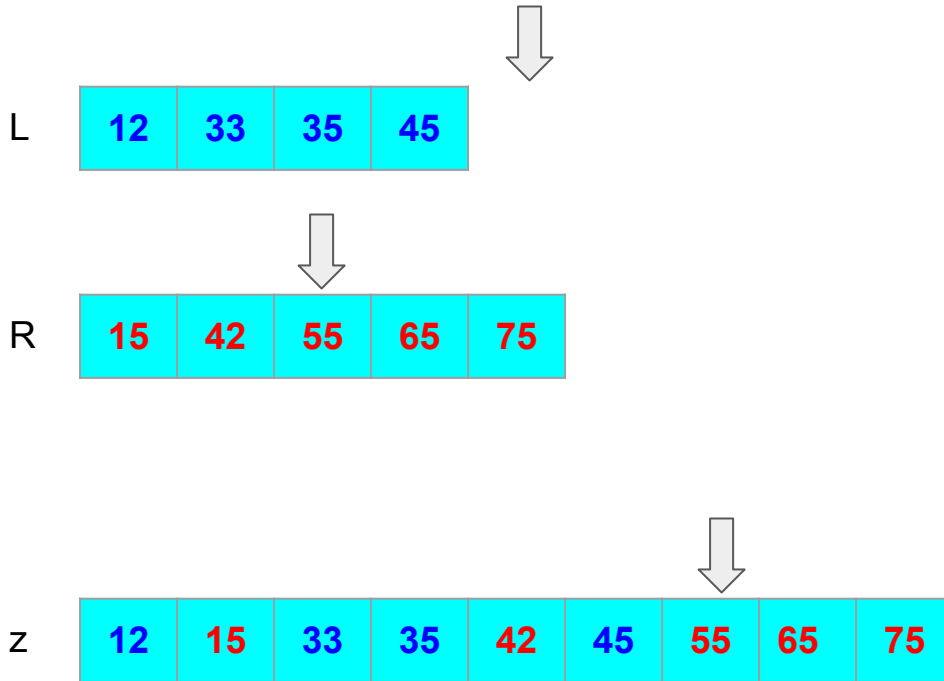
```
while iL<=length(L) && iR<=length(R)
  if L(iL) <= R(iR)
    z(iz) = L(iL);
    Increment iL and iz;
  else
    z(iz) = R(iR);
    Increment iR and iz;
  end
end
```

How can we merge two sorted arrays?



```
while iL<=length(L) && iR<=length(R)
  if L(iL) <= R(iR)
    z(iz) = L(iL);
    Increment iL and iz;
  else
    z(iz) = R(iR);
    Increment iR and iz;
  end
end
% if iR <= length(R), put in z
% if iL <= length(L), put in z
```

How can we merge two sorted arrays?



```
while iL<=length(L) && iR<=length(R)
  if L(iL) <= R(iR)
    z(iz) = L(iL);
    Increment iL and iz;
  else
    z(iz) = R(iR);
    Increment iR and iz;
  end
end
% if iR <= length(R), put in z
% if iL <= length(L), put in z
```

How can we merge two sorted arrays

```
function z = merge(L,R)
% Merge two sorted arrays L and R
nL = length(L); nR = length(R);
z = zeros(1, nL+nR);
iL = 1; iR = 1; iz = 1;
while iL<=nL && iR<=nR
    if L(iL) <= R(iR)
        z(iz)= L(iL); iL=iL+1; iz=iz+1;
    else
        z(iz)= R(iR); iR=iR+1; iz=iz+1;
    end
end

% if iL <= nL, put rest of L in z

% if iR <= nR, put rest of R in z
```

How can we merge two sorted arrays

```
function z = merge(L,R)
% Merge two sorted arrays L and R
nL = length(L); nR = length(R);
z = zeros(1, nL+nR);
iL = 1; iR = 1; iz = 1;
while iL<=nL && iR<=nR
    if L(iL) <= R(iR)
        z(iz)= L(iL); iL=iL+1; iz=iz+1;
    else
        z(iz)= R(iR); iR=iR+1; iz=iz+1;
    end
end
while iL<=nL % copy remaining L-values
    z(iz)= L(iL); iL=iL+1; iz=iz+1;
end
while iR<=nR % copy remaining R-values
    z(iz)= R(iR); iR=iR+1; iz=iz+1;
end
```

Merge sort: both codes together

```
function y = mergeSort(x)
% x is a vector. y is a vector
% consisting of the values in x
% sorted from smallest to largest.
n = length(x);
if n == 1 || n == 0
    y = x;
else
    m = floor(n/2);
    yL = mergeSort(x(1:m));
    yR = mergeSort(x(m+1:n));
    % merge sorted yL and yR
    y = merge(yL,yR);
end
```

```
function z = merge(L,R)
% Merge two sorted arrays L and R
nL = length(L); nR = length(R);
z = zeros(1, nL+nR);
iL = 1; iR = 1; iz = 1;
while iL<=nL && iR<=nR
    if L(iL) <= R(iR)
        z(iz)= L(iL); iL=iL+1; iz=iz+1;
    else
        z(iz)= R(iR); iR=iR+1; iz=iz+1;
    end
end
while iL<=nL % copy remaining L-values
    z(iz)= L(iL); iL=iL+1; iz=iz+1;
end
while iR<=nR % copy remaining R-values
    z(iz)= R(iR); iR=iR+1; iz=iz+1;
end
```


Merge sort: both codes together

```
function y = mergeSort(x)
% x is a vector. y is a vector
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    m = floor(n/2);
    yL = mergeSort(x(1:m));
    yR = mergeSort(x(m+1:n));
    % merge sorted yL and yR
    y = merge(yL,yR);
end
```

```
function z = merge(L,R)
% Merge two sorted arrays L and R
nL = length(L); nR = length(R);
z = zeros(1,nL+nR);
iL = 1;
iR = 1;
iz = 1;
while iL <= nL
    z(iz) = L(iL); iL = iL + 1; iz = iz + 1;
while iR <= nR
    z(iz) = R(iR); iR = iR + 1; iz = iz + 1;
while iL <= nL % copy remaining L-values
    z(iz) = L(iL); iL = iL + 1; iz = iz + 1;
while iR <= nR % copy remaining R-values
    z(iz) = R(iR); iR = iR + 1; iz = iz + 1;
end
```

Challenge question:

When I call mergeSort(x)
when x has length 17,
what is the maximum
number of mergeSort **call
frames** that will be open
at one time?

We have a sorted array, now what?

6 10 15 23 29 31 32 45 47 52

We can still use linear search to search for items in a sorted list.

When you search through a phone book, you do something a little more efficient than linear search...

wide at SuperPages.com

195 Car C

17 566-1282	Cartage New England Inc 26 Allen Ln Ipswich 01938.....	978 356-9960	Carter F 24 Hillcock Res 02131.....	617 327-1105	Carter Nella E 333 Marchants Av Bos 02215.....	617 267-6483
81 447-4101	Cartagema Lydia 18 Jewett Res 02131.....	617 323-7639	Faye & Ricky 157 Columbus Av Bos 02116.....	617 437-7331	Nicholas S F 115 Randolph Av MI 02186.....	617 698-5307
80 257-9981	Cartagema Avith 9 Baconrft Rm 02119.....	617 442-9780	Francis S 134 Temple W Rm 02132.....	617 323-6781	Nick 21 Fairfield Bos 02116.....	617 267-5222
17 566-1282	B Hyd 02136.....	617 361-5253	Franklin & Anne 221 Mt Auburn Cam 02138.....	617 354-0798	Nick & Debbi 136 Herrick Rd Newton 02459.....	617 527-0480
17 364-5188	Jessica 50 Decatur Cha 02129.....	617 241-0152	Fred 42 Haverford Jam 02130.....	617 524-3078	Nicole 136 Herrick Rd Newton 02459.....	617 698-0713
361-0380	Lucilla 174 Harvard Cam 02139.....	617 491-5621	Fred 96 Hinckley Rd MI 02186.....	617 698-1343	Norman G 38 Chickawabot Dor 02122.....	617 822-1203
17 566-4548	M 95 Rowe Res 02131.....	617 323-9713	G & R 8 Verdun Dor 02124.....	617 436-8906	P 94 Crestwood Pk Res 02121.....	617 427-4754
17 628-8248	Melvin 301 Green Cam 02139.....	617 576-1061	G T 27 Franklin Av Som 02145.....	617 623-7121	P E 501 E Sixth St Bos 02127.....	617 268-4213
17 445-5116	Carte Nicholas 18 Appleton Boston 02116.....	617 695-6996	Gayle 25 Frances Dor 02124.....	617 825-0322	P L 44 Hutchings Res 02121.....	617 427-9170
17 822-2982	Cartegena O 4 Milford Bos 02118.....	617 338-8219	George 125 Nashua Bos 02134.....	617 367-9548	P R 91 Byrner Jam 02130.....	617 983-8692
17 427-5712	Carten Thos J Sr & Claire 1 Paradise Rd MI 02186.....	617 698-6163	Carter Halliday Associate 107 S Street Bos 02111.....	617 456-1689	Paul & Constance 114 Anawan Av W Rm 02132.....	617 325-2036
17 569-2698	Thomas & Kathleen 50 Thompson Ln MI 02186.....	617 696-6919	Carter Harry F 26 Rungg Birk Rd W Rm 02132.....	617 325-5465	Paul E 501 E Sixth St Bos 02127.....	617 268-4546
17 667-5190	Cartier A Res 02131.....	617 327-2257	Carter Hide Co Inc 146 Summer Bos 02110.....	617 542-7987	Paul M 27 Union Bri 02135.....	617 787-2115
17 569-1417	A Roxbury 1750 Centre St MI 02186.....	617 442-5230	Carter Hilary 61 Harvey Cam 02140.....	617 876-2750	Carter Pile Driving Inc 17 Beaver Ct Frankington 02102.....	Wellesley Tello 781 235-9488
17 338-9110	A 31 Redhne Wly Roxbury 02119.....	617 442-1219	Horace 241 Walnut Av Roxbury 02119.....	617 442-5307	Carter Prudence 46 Franklin Watertown 02172.....	617 393-3782
17 825-9195	A 288 Pitman Av Cambridge 02139.....	617 492-4174	Howard Jr 26 Weber Dme Rm 02118.....	617 445-5552	Prudence 46 Franklin Watertown 02172.....	617 926-7063
17 296-1593	A M 255 Marchants Av Bos 02215.....	617 286-7153	J Cam J 15 Chatham Bro 02446.....	617 232-7990	Reginald 106 Braintree Dorchester 02215.....	617 541-2843
17 670-2078	Adams 361 Centre St MI 02186.....	617 698-9074	J 518 Harvard Bro 02446.....	617 730-9483	Renee & Andrew 30 Walnut Bos 02108.....	617 720-3765
17 623-9001	Alice 106 Kilmarock Bos 02215.....	617 425-0193	J 775 Vha Pkwy West Roxbury 02132.....	617 323-5574	Carter Rice Dowd Bulley Dutton Publishing 163 Main Wilmington 01887 Toll Free-Old '1' & Thes.....	800 638-1671
17 296-4725	Andrew F 62 Vinal Av Som 02143.....	617 625-7623	Carter J Jacques MD 1 Brookline Pl Bro 02446.....	617 735-8787	Cost Svc Industrial Prod 612 Main Wilmington Toll Free-Old '1' & Thes.....	800 619-7447
17 542-1521	Cartier Anne MD 1101 Beacon Bro 02446.....	617 739-1022	Carter J M 1410 Columbia Rd S Bos 02127.....	617 464-1040	Cost Svc Printing 612 Main Wilmington Toll Free-Old '1' & Thes.....	800 648-7447
17 364-5232	Carter Athens 272 Newbury Boston 02116.....	617 536-6329	Carter J M Ornamental Ironworks Call.....	Pembroke Tello-617 436-5353	Headquarters 612 Main Wilmington 01887 Call.....	978 988-7447
17 541-5649	Carter Barbara L MD Turbo-New England Medical Center Bos 02111 Call.....	617 636-0051	Carter J Veal Co 48 Newmarket Sq Res 02118.....	617 442-1775	Ingalls Crane 163 Main Wilmington 01887 Toll Free-Old '1' & Thes.....	800 638-1673
17 739-2662	Carter Becky Bos 02114.....	617 523-4368	Carter James 1573 Cambridge St Cam 02138.....	617 492-1214	Carter Richard 1679 Commwllth Av Brights 02215.....	617 987-0836
17 879-0030	Bernard J 112 Goddstone E Bos 02128.....	617 567-3430	James 182 Fisher Av Roxbury 02130.....	617 739-2193	Richard A 97 Mt Vernon Bos 02186.....	617 566-7293
17 541-3948	Bithiah 25 Medway Dor 02124.....	617 298-8713	James 17 Gold Star Rd Cambridge 02140.....	617 876-8841	Carter Richard A MD 170 Commwllth Av Bos 02116.....	617 267-0710
17 569-4119	Blake 26 Mt Vernon Bos 02186.....	617 367-9931	Jane L 14 Rosaberry Rd Mat 02126.....	617 361-0773	Carter Richard K 157 Mercer S Bos 02127.....	617 268-0448
800 569-8782	Carter Broadcasting Co 20 Park Plt Bos 02116.....	617 423-0210	Jeffrey 41 Norton Av Bos 02116.....	617 426-5994	Robert L 175 Richtable Av Cam 02140.....	617 864-1535
	Carter & Burgess Consultants Inc 23 East St Cam 02141.....	617 225-0200	John 11 Mansfield Rd 02134.....	617 987-2163	Roger 150 St Botolph Bos 02115.....	617 424-6148
	Carter C 2000 Commwllth Av Bri 02135.....	617 782-2118	John 327 Summer Bos 02110.....	617 423-4334	Roy 44 Concord Av Cam 02138.....	617 491-6115
	C 298 Foywood Av East Boston 02268.....	617 569-1545	John 40 Westwind Rd Dor 02125.....	617 282-1235	Royce 18 Seminary Cha 02129.....	617 241-4418
	C 359 Harvard Cam 02138.....	617 491-4822	June O 329 A Summit Av Bri 02135.....	617 734-6109		
	C 610 Walk Hill Mat 02126.....	617 296-6392	K 38 Browning Av Dorchester 02124.....	617 265-9456		
	C & M 43 Burroughs Jam 02136.....	617 524-9558	K 17 Esmond Dorchester 02123.....	617 282-1593		

Key idea of “phone book search”: repeated halving

To find the page containing Benito Antonio Martínez Ocasio’s number...

While (phone book is longer than 1 page)

 Open to the middle page

 If “Ocasio” comes before the first entry on the current page

 Rip and throw away the second half

 Else

 Rip and throw away the first half

 End

end

What happens to the phone book length?

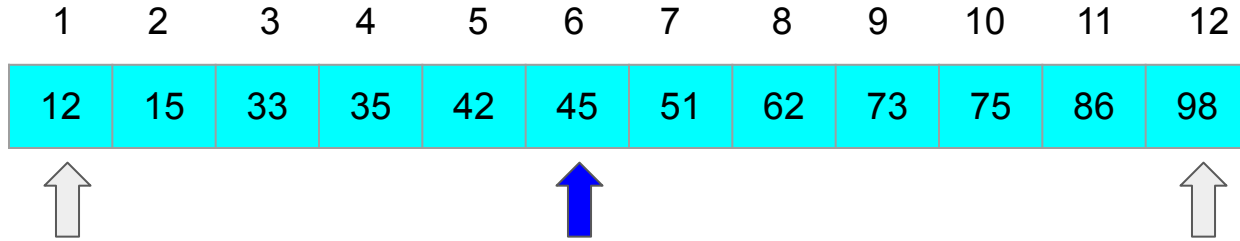
Original:	3000	pages
After 1 rip:	1500	pages
After 2 rips:	750	pages
After 3 rips:	375	pages
After 4 rips:	188	pages
After 5 rips:	94	pages
⋮		
After 12 rips:	1	page

Binary search

- Repeatedly halving the size of the “search space” is the main idea behind the method of binary search
- An item in a sorted array of length n can be located with just $\log_2 n$ comparisons
 - Where does this \log come from? Mathematics! Take a data structures and algorithms class to find out how!
- Time savings are significant!

n	$\log_2 n$
100	7
1000	10
10000	13

Binary search: target = 70



L	1
R	12
Mid	6

Since $v(\text{mid}) < \text{target}$, we know that we can throw away everything before index mid

Binary search: target = 70

1	2	3	4	5	6	7	8	9	10	11	12
12	15	33	35	42	45	51	62	73	75	86	98

↑ ↑ ↑

L	6
R	9
Mid	7

Since $v(\text{mid}) < \text{target}$, we know that we can throw away everything before the index mid

Binary search: target = 70

1	2	3	4	5	6	7	8	9	10	11	12
12	15	33	35	42	45	51	62	73	75	86	98

↑ ↑ ↑

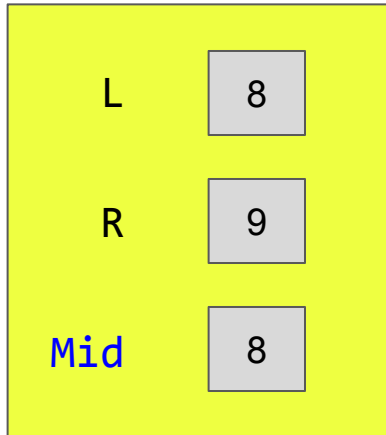
L	7
R	9
Mid	8

Since $v(\text{mid}) < \text{target}$, we know that we can throw away everything before the index mid

Binary search: target = 70

1	2	3	4	5	6	7	8	9	10	11	12
12	15	33	35	42	45	51	62	73	75	86	98

↑ ↑



Done because $L - R == 1$

```
function L = binarySearch(x, v)
% Find position after which to insert x. v is sorted in ascending
% order. L is the index such that v(L) <= x < v(L+1);
% L=0 if x<v(1). If x>v(end), L=length(v) but x~v(L).
```

```
function L = binarySearch(x, v)
% Find position after which to insert x. v is sorted in ascending
% order. L is the index such that v(L) <= x < v(L+1);
% L=0 if x<v(1). If x>v(end), L=length(v) but x~v(L).

% Keep halving your search space [L, R] until R-L is 1,
% always keeping v(L) <= x < v(R)
while R ~= L+1
    m = floor((L+R)/2); % middle of search window
    if v(m) <= x
        % get rid of everything in left half
    else
        % get rid of everything in the right half
    end
end
end
```

```
function L = binarySearch(x, v)
% Find position after which to insert x. v is sorted in ascending
% order. L is the index such that v(L) <= x < v(L+1);
% L=0 if x<v(1). If x>v(end), L=length(v) but x~v(L).

% Keep halving your search space [L, R] until R-L is 1,
% always keeping v(L) <= x < v(R)
while R ~ = L+1
    m = floor((L+R)/2); % middle of search window
    if v(m) <= x
        L = m;
    else
        R = m;
    end
end
end
```

```

function L = binarySearch(x, v)
% Find position after which to insert x. v is sorted in ascending
% order. L is the index such that v(L) <= x < v(L+1);
% L=0 if x<v(1). If x>v(end), L=length(v) but x~v(L).

% Maintain a search window [L..R] such that v(L)<=x<v(R).
% Since x may be outside of the range of v, initially set ...
L = 0; R = length(v)+1;

% Keep halving your search space [L, R] until R-L is 1,
% always keeping v(L) <= x < v(R)
while R ~ = L+1
    m = floor((L+R)/2); % middle of search window
    if v(m) <= x
        L = m;
    else
        R = m;
    end
end
end

```

Check out binarySearch.m and showBinarySearch.m on the course website!